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THESIS

COLLATERAL READING FOR THE HIGH SCHOOL MATHEMATICS TEACHER

SUBMITTED BY

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I. Introduction.

A little more than sixty-five years ago, Herbert Spencer gave "complete living" as the aim of the English secondary schools. In the analysis of "those activities which by securing the necessaries of life indirectly minister to self preservation" (11) he includes the teaching of arithmetic. In correspondence with Spencer the Commission on the Reorganization of Secondary Education gives the "command of fundamental processes" (11) including arithmetical computations as one of the seven aims. Spencer also adds that "further training in all those scientific and mathematical principles which underlie the occupations of the manufacturer, builder, farmer, and merchant should be accorded their rightful place in the time set aside for instruction" (11). Despite the fact that the present tendency, in theory, is to devote less time to mathematics in the secondary school, yet the subject cannot be thrown out recklessly. Mathematics is like sunshine. It permeates our life (38) says Schwab. It is the foundation of all science. It exemplifies especially well certain important modes of thought. (45).

It is customary to read into the life story of a person who stands out in our estimation, to peruse over details. When one book is finished, more are gathered. Mathematics cannot be compared to this person in the physical sense but its rays of interest do reflect into the lives of many. Being such a

(11) Douglass. Secondary Education. page 327.
(11) Douglass. Secondary Education. page 331.
(11) Douglass. Secondary Education. page 328.
(38) Schwab. Mathematics and Sunshine. The Mathematics Teacher. MAY 1928
(45) J.W.A.Young. Teaching of Mathematics in the Elementary and Secondary School. page 17. ~~May 1928~~.

vast subject with its many intriguing problems and equations, it has an exceedingly interesting history.

II Collateral Reading.

1. Histories of Mathematics.
2. Teaching of Mathematics.

   Causes of Inefficiency in the Teaching of Mathematics and Suggestions with Respect to

   (a) Algebra

   (b) Geometry
3. Periodicals
4. Recreational Books

1. Thus, is it too great a demand to ask the Mathematics High School Teacher to have a reasonable historical knowledge of this great science which he teaches from day to day?

One of the foremost histories of mathematics is that of David Eugene Smith.(39). Its purpose is that of supplying students with a usable textbook on the history of elementary mathematics.

Volume one is a general survey beginning with prehistoric mathematics. Then the growth of mathematics is divided into periods, for example, 1000 B.C. to 300 B.C., 300 B.C. to 500 A.D. From the sixteenth century on, the history of mathematics is dealt with by centuries.

Volume two is a topical survey. The history of the development of the various topics in mathematics, arithmetic, algebra, geometry etc. are fully and interestingly described.

These two volumes may be used with convenience. A chronological table is given with the date of birth added to each name. By means of four types of print, the importance of a mathematician or fact can be determined. In the index only the important references are given and not as in Tropfke (43) and Cantor (9) the page given every time a name is mentioned. This is a very compact and reliable book.

W.W.Rouse Ball's (4) account of history like Smith's is a detailed work. Both histories are valuable but this is an older book having been revised four times since 1888. This one volume contains a general and topical survey at once. The Greek period, the mathematics of the Middle Ages and the Renaissance, and the Modern period represent the history of mathematics as divided into three definite periods. This

book sets forth a short and popular account of facts for those who are interested and yet haven't ample time to go deeply into the history of mathematics.

Cajori's history (7) is similar to Ball's in importance. The first part of the book deals with the history of mathematics according to countries; the second, according to topics.

For the development of mathematics in the United States (8), Cajori offers a very good book. From Colonial times to the present a valuable account is rendered. Historical essays on infinite series, fundamentals of Algebra, Circle Squares, and difference between Napier's and Natural Logarithms complete the story.

The translation of Dr. Karl Finks(14) Geschichte der Elementär-Mathematik by W.W.Beman is a brief account for the student and teacher of mathematics. A topical survey, in general, with a racial background. The study of geometry is developed by referring to the early works of the Egyptians, Babylonians, and Greeks. Then the Roman, Hindu, Chinese, and Arabian contributions are alluded to, summing with the work of Gerbert through Descartes to the present time. The biographical notes are prize-worthy as well as the simple descriptions of a list of mathematicians.

For an historical introduction to mathematical literature Miller's book (30) is the only one of its kind. It is not intended to be a pure history of mathematics as Smith's, Ball's, or Cajori's are, yet it contains a topical survey. The author presents a critical point of view to mathematical literature in general. The origin and work of the mathematical societies

of the continent and United States are described. Under geometry a few fundamental notions are spoken of in detail but a history of the development of geometry, in general form, is not discussed. A list of twenty-five prominent deceased mathematicians ends this critical discussion.

To make use of the vast account in Cantor's (9) volumes, a reading knowledge of German is necessary. These volumes offer detailed and concise informational material. The same description may be applied to Tropfke's (43) volumes. In both works, the mathematical problems are attended by the history of their development.

For further interest in mathematicians one may turn to Memorabilia Mathematica (31). The rest of this book is given to quotations and definitions of mathematics.

As we know, the Greeks laid a very good foundation to the science of mathematics with respect to geometry. It is, therefore, not unusual to find two very good books dealing with Greek mathematics. Allman's (2) work, first a series of lectures, was later published in book form describing the life and accomplishments of all the great Greek mathematicians from Thales to Euclid. Heath(17) offers two volumes with more detailed information as to particular problems and life histories of Greek mathematicians. Much improvement in knowledge may be derived from the use of these two books.

For particular emphasis of phases of Greek mathematics one can refer to Apollonius of Perga (20) and his work on the conics, to the works of Archimedes (18) or to the thirteen books of Euclid's Elements.(19).

At times the life history (15) and not the mathematical accomplishments of these early men are sought. Diogenes Laertius (25), a biographer of eminent philosophers, not particularly interested in mathematics, gives a vivid portrayal and tends to scatter many vague notions which one gathers from merely considering one side of a person's life.

Not until the nineteenth and twentieth centuries was a key perfected to understand the mysteries baked into the wax tablets of the inhabitants of the Tigris and Euphrates Valley(21). Accomplishing little in geometry, the Babylonians, nevertheless, had developed a business arithmetic in a sexagesimal system. We are apt to think in terms of the Greeks only as being great mathematicians. Perhaps, as a group, they offered more but their work was greatly influenced by that of the Egyptians and Babylonians.

Likewise, in the twentieth century, was the work of Ahmes revealed. The Rhind Mathematical Papyrus (34) contains a complete history of Egyptian mathematics, Ahme's contributions, and illustrations of the calculation system. For further information regarding Ahmes, one may refer to the Story of Mathematics.(26).

During the Greek development, mathematics was also making headway in China and Japan (29). The Chinese and Japanese claim their science was taken over to western Europe. There may be claim for slight transfer but, on the whole, each developed independently of the other.

Indian mathematics (24) is considered as a minor phase in the development of mathematics. It offers an exceedingly

interesting story showing Greek and Chinese influence.

An exceedingly interesting and valuable book is that called "Makers of Science" (16). Many of the mathematicians from the seventeenth century to more modern times were interested in science as well. The development of Physics and Astronomy vied with that of mathematics during this period. To secure complete information about mathematics it is necessary to refer to books of science.

2. What are the causes of inefficiency in the teaching of mathematics? "The inefficiency of teaching mathematics is not due to the subject but to the same general causes that are at work in the teaching of any subject," (1)says Schultze. Schools have been too interested in producing spectacular results. Too much time has been foolishly spent, merely for the sake of show and appearance for the outside world. Examinations have been made a central fact. "When they assume this competitive character, ill effects result for they are not proper gauges of the quality of the school." Then there has been a tendency of offering too much mental food, more than properly could be assimilated. The result is mental indigestion. There are many excelling students whose memories rather than reasoning powers have been trained. Finally, many schools have been opposed to the "true mathematical spirit". Good mathematics teaching has not been understood. Mathematics was taught for imparting knowledge and suffered thereby." (37).

"Failure in mathematics is due to unatractiveness."(42). There is constant talk about the need of new educational

theories with respect to this scientific age. Why not apply the lauded value of the motion picture to mathematics with respect to demonstration? "A carefully prepared and successfully performed demonstration is of more value than many a verbal description".

Another cause is that pupils do not know how to study. They learn material by memorizing, not by thinking. Supervised study (2) can aid in overcoming this fault. It affords individual instruction, thereby bringing the teacher in close contact with the pupil so he will not demand too much or too little of him.

Many a class in mathematics lacks the appreciation characteristic. No wonder the subject does not appeal and the consequent result is failure. It is necessary, then, for the teacher to know how each subject in mathematics has been taught, why it is taught, and how it should be developed. If this course is followed "elementary mathematics will take on a new reality for those who study it." (39).

How many teachers, until recently, have had specific training in mathematics? Very few, according to Young (46). The mathematic teacher of Germany has a vigorous training. England and America are now following her example.

Has difficulty arisen in carrying the subject of mathematics too far? The present trend is to teach mathematics for the practical value. This would limit the subject to arithmetic. "But," says the author (46), "we use only the common elements of English, History etc. and yet go further in each subject." That cannot be a cause. The chief value of study

of mathematics is "mathematics as a mode of thought." If students have had difficulty, it is not because they had no talent for it, but because of past teaching or they have memorized rather than thought through the subject.

The school has been inefficient not only in mathematics but in other subjects as well in not providing for the student who leaves school at the end of the ninth grade. "The most obvious points of contact between the school and the Junior High School pupil are the graphical method and use of formulae" (12). As soon as the school meets the future needs of the pupil, in mathematics, with respect to his life in a democracy, the question of inefficiency will be lessened.

"The twentieth century is to make mathematics in the elementary and secondary school more useful and educative" (6). A new mathematics is necessary and desirable and it may be scientifically taught. The old form of mathematics has to be discarded to meet the present needs.

To meet the present needs the National Committee on Mathematical Requirements has issued a report which is necessary for every teacher to know about (45).

2a. Algebra has caused so much trouble. Why? Algebra has not been organized to offer practical results. Textbooks have confined themselves to the deductive method. Those textbooks attempting to combine "former theoretical" and "present practical" methods have not been "developing sound habits of quantitative thinking". (28).

If students are properly motivated, if they can see the practical side of the problem, they will be induced to study.

"The best foundation of mathematics teaching is the constant use of inductive and deductive methods with a clear understanding of the learning process." Both methods must be used interchangeably.

Problems have always been a bone of contention. If problems are analyzed in a diagrammatic way showing the relations among quantities, no difficulties will arise. If pupils realize that all problems are solved by a "characteristic formula", problem solving will be greatly simplified. "Every problem solved by means of the function idea will strengthen the pupils grasp of subject as a whole."(28).

Another suggestion, is that, after the fundamentals are learned the subject will simplify itself. Therefore, stress should be placed on the fundamentals.(28).

"Mathematics should have contact with the outside world in time and space, and with itself.(32). It should develop the mind and give mental training." It has been customary to teach it by parts and not as a whole. The difficulty in algebra is due to the fact of being kept by itself. Algebra should take a good stand in arithmetic. By means of algebra, trignometic and geometric formulas may be placed into the study of arithmetic. Diagrams, in profusion, are suggested for relieving the strain of abstractness.

2(b). Geometry may be taught more efficiently if the purpose of teaching it is realized. Should it be taught for its utilitarian value and to train the mind? No. "We study it because we derive pleasure from contact with a great and ancient body of learning that has occupied the attention of master

minds during the thousands of years in which it has been perfected and we are uplifted by it."(41).

"The appreciation of literary and artistic beauty has of late received increasing recognition. The contemplation of unassailable mental structures such as are found in mathematics cannot but raise the ideal of perfection different in nature from those found in the more emotional creations of literature and art." (10).

For its own sake, then, geometry has the right to be in the education of every human being.

The method and class procedure in teaching geometry can be improved upon. "The mastery of the proof of a theorem should suffice at first."(46). "The polishing can be done later. Many of the difficult theorems should be skipped until a more advantageous time or, if it is necessary to make use of them, their proofs should be taken for granted. Rigorous proofs should not be learned but found by the class with the teacher in discussion or by the pupil alone."(46).

In like manner, many textbooks attempt to "present preliminary propositions rigorously."(37). This is foolish for it is a waste of time and space. In giving proofs there is no need of pupils adhering to every minor detail. The reasoning powers should be strengthened not the memorizing powers. Exercises should assume a good part in the study of geometry for pupils take delight in working out something for themselves not always following other people's ideas.

3. Periodicals more than histories of mathematics or even

books on pedagogy do bring the teacher in closer contact with his everyday classroom problems. The articles contained in these magazines do strive to meet the needs of all.

For high school teachers, "The Mathematics Teacher" is perhaps the most valuable. It is the official journal of the National Council of Teachers of Mathematics. It appears monthly from October to May. Its purposes are to:-

1. Create and maintain interest in the teaching of mathematics.

2. Keep the values of mathematics before the educated world.

3. Help the young and inexperienced teacher to become a good teacher.

4. Improve teachers in service by making them better teachers.

5. Raise the general level of instruction in mathematics.

Each year a "Year Book" is published by this same society, dwelling upon the important topics of the year. So far, three year books have been published.

"The American Mathematical Monthly" is devoted to the interests of collegiate mathematics containing

1. Problem department

2. News items

3. Book reviews

This journal is published by the Mathematical Association of America. This is a very popular society. Unfortunately, there is little in the line of pedagogy discussed at its meetings.

"School Science and Mathematics" is a journal for all science and mathematics teachers. Thirteen departments are

represented of which eleven are sciences. The other two are mathematics and mathematical problems.

"The Bulletin" might also be mentioned, published by the American Mathematical Society. This is a research society composed of graduate school teachers. "The Bulletin" contains general news, records of meetings, and brief sketches of written papers. Only the important articles of research work are entirely published. Consequently, there are few articles which are directly useful for the secondary school teacher.

4. Recreational books in mathematics present an ideal setting for amusement. It is quite interesting to note the number and types of these books; one, in particular, dating back to 1775. The recreational suggestions in these books are for teachers, pupils, and the public in general. Many a pleasant hour may be spent in unfolding their mysteries.

> "In Mathematics he was greater
> Than Tycho Broke or Erro Pater:
> For he, by geometric scale,
> Could take the size of pots of ale;
> Resolve, by sines and tangents straight
> If bread or butter wanted weight;
> And wisely tell what hour of the day
> The clock does strike, by algebra."
> Butler's Hudibras.

Thus is the theme of this great puzzle book expressed.(12). Puzzle after puzzle, arithmetical or geometrical, appears. Patchwork quilts, chessboard tricks, mazes, and Greek cross puzzles will delight the reader. Solutions at the end will settle many a dispute.

For more serious study, Recreations in Mathematics and Natural Philosophy (33) will supply the need. This is a most

complete book. It is not intended to be a text-book but gives that appearance. One could spend many an hour perusing its pages. It is advantageous for secondary school work. Our modern text-books on natural philosophy are more up-to-date but lack the accompaning historical facts which this work offers. This book was first written by Ozanam in the seventeenth century. In the eighteenth century Montucla edited this text with additions. Later Charles Hutton translated the work into English. In 1890 Edward Riddle published a new and revised edition.

In the geometry class, an appreciation lesson may profitly be developed by giving geometric exercises in paper folding (35), squaring the circle (22), or dicussing the fourth dimension (36). Every secondary school pupil enjoys working out a geometrical or mechanical recreational problem (5) just for the fun of it.

What pleasure, at times, to remove oneself, mentally, from this sphere and visit another one where men take the shape of squares, pentagons, and hexagons; the highest type of nobility, circles; and women and clergy are represented by straight lines. Strange dwellings in this land, all consisting of five sides each with two opposite doors, one for the men and one for the women. It must be remembered that these women are straight lines and it is not always possible to see them. To avoid accidents, necessitates two doors. These people move about like shadows, neither arising above or below the plane,for this is Flatland.(1).

Perhaps floating from one sphere to another may incite

interest in astronomy or astrology (23), its forerunner, which played an important part in the lives of the Egyptians and Greeks.

Magic squares and cubes (3),(36), have made quite a place for themselves in mathematical literature from prehistoric times on. Their origin was in China but the Arabs and Hebrews used them also. There are many possibilities of adding, subtracting, multiplying, and dividing squares (12) and forming squares of four, five and six blocks etc.

This little book (27) states its own aim in "affording recreation for an idle hour and inciting young students to further mathematical inquiries." It offers Greek and Arabian notations, Roman numerations, early arithmetic in England, methods of computing π, and historical construction of pyramids of Egypt ending with overlooked fallacies in higher mathematics.

For a diversion which demands new thoughts, every day, this ideal book (44) will suit the need of the most fastidious. Each page contains something new. Such a scrap book of elementary mathematics is a veritable pleasure.

III Conclusion

1. Plan for using Supplementary Material in

    a. Seventh grade

    b. Eighth grade

    c. Ninth grade

    d. Tenth grade

    e. Eleventh grade

    f. Twelfth grade

2. An Illustrative Unit in Appreciation

    Newton's Law of Gravitation for High School Students.

1. For using supplementary and approximate materials in the high school course I am submitting a plan whereby references are given for appreciation units in grades seven to twelve inclusive. This is by no means an exhaustive plan but acts merely as a suggestion of what can be done with the proposed collateral reading for the mathematics high school teacher.

As an illustration an appreciation unit has been appended on Newton's Law of Gravitation for High School Students.

(a) Seventh grade - Introduction to Algebra

1. Forming the correct American flag 6, (P.12,79)
2. Use of right triangles 13,(P.40), 33 (P.162.)
3. Use of formulas 6 (P.37,83), 13(P.14).
4. Algebra in Egypt, China, Greece etc. 39(P.379)II
5. Negative numbers 37 (P.302), 46(P.315,245,300).
6. Graphic problems 37 (P.335, 110).

(b) Eighth grade - Intuitive Geometry

1. Early methods of measuring distance 39 II (P.344).
2. Barrels of honey 12 (P.111).
3. Mazes 12 (P.127).
4. Sense of direction (flatland) 1.
5. First geometry book 26 (P.27).
6. In days of pyramids 26(P.18), 27(P.51).

(c) Ninth grade - Algebra

1. Problem solving 28 (Chapters 5,7)
2. Magic squares 27 (P.39), 36 (P.39), 3.

3. Newton's problem expressing algebraic language 46 (P.313).
4. Path of projectile - quadratics 6 (P.103).
5. Use of equations 13 (P.17).
6. Use of colored diagrams 32 (Vol.I. P.208, 210).

(d) Tenth grade - Plane Geometry

1. Division of circle into three parts 27 (P.48) (Squaring circle 22, 39 II (P.302), 36 (P.112)).
2. Value of π 6 (P.51), 27 (P.49), 26 (P.66) 39II(P.307)
3. Trisection of angle 27 (P.49), 5 (P.291) 33 (P.146) 37 (P.259).
4. Square puzzles (fitting areas together) 12 (P.35).
5. To make a straight line very nearly equal arc of circle 33(P.152).
6. To find the value of an angle by means of a compass only (no part of circle given) 33 (P.148).

(e) Eleventh grade - Intermediate Algebra including Analytic Geometry

1. Formation of Magic Squares 27 (P.39), 36 (P.44),3.
2. Hints for graphic solution of equations 27 (P.78, 84 13 (P.41).
3. To describe a true ellipse geometrically 33 (P.137)
4. To make the same body pass through a circle, ellipse, and square 33 (P.149).
5. Practical problems using trignometric functions 32 (P.121, Vol.1.)
6. Omar Khayyam's solution of quadratics etc. 39 II (P.447).

(f) Twelfth grade - Trigonometry - Solid Geometry

1. Making of a sun dial (T) 27 (P.112).
2. Hints on teaching trignometry 37 (P.353).
3. Trigonometric fallacy in proving angles of a triangle equal one another 27 (P.66).

1. Spiral problem - Jack and Beanstock 12 (P.145).
2. Duplication of the cube 5 (P.285), 33 (P.147).
3. Quadrature of circle 5 (P.293), 39 II (P.305).
4. Form in which bees construct their combs 33 (P.141).

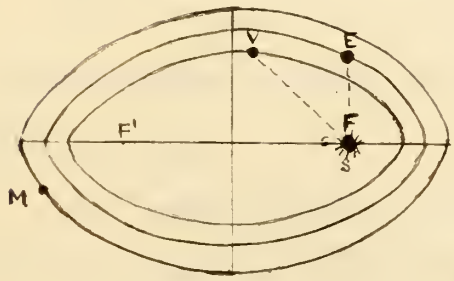
2. Newton's Law of Gravitation (for High School Students.)

In 1609 the German astronomer Kepler, having made a careful study of Tycho Brahe's observations, announced the following laws:-

1. The orbits of the planets are ellipses having the sun at one focus.

2. The area swept over per hour by the radius joining sun and planet is the same in all parts of the planet's orbit.

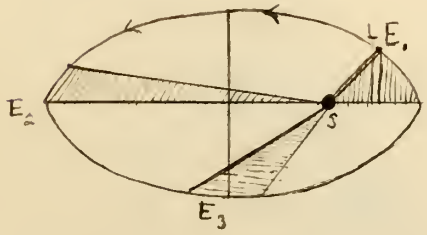
3. The cubes of the mean distances of any planets from the sun are proportional to the squares of the periodic times.



V Venus
E Earth
M Mars
S Sun
F and F' focii.

Just what do these laws mean? Let us look at the first one. All planets such as Venus, the Earth and Mars travel in a curve around the Sun which is called an ellipse. The sun is at one focus of the ellipse. Before Kepler's time it was thought that all planets travelled in circles.

The second law says that no matter what position the Earth (or any planet) takes in its path around the Sun, the line from the Sun to the Earth sweeps over equal areas in equal intervals of time. That is, the



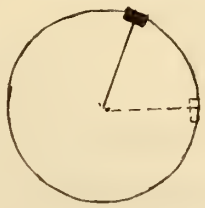
area swept over by the line SE_1 is equal to that of SE_2 and to that of SE_3 if in each case the time was the same. By looking at the crude diagram the areas appear to be equal.

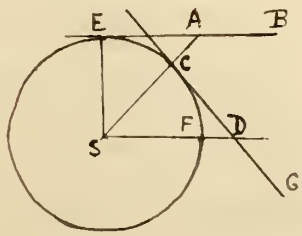
The third law states that $\frac{SE_1^3}{T^2}$ is proportional for the planets, that is, the square of the time it takes a planet starting at L to go all the way around the ellipse and back to L is proportional as the cube of its mean distance from the sun or the third law states that $\frac{SE^3}{T_1^2} = \frac{SM^3}{T_2^2}$ where T_1 = period of Earth and T_2 = period of Mars. Calling the mean distance SE_1, the third law may be written as $\frac{r^3}{T^2}$ where T is the period or time spent in going around the elliptic curve once.

Why should this law be true, asked Newton, the English mathematician? Kepler had merely stated his laws without proving them.

For simplicity's sake let us consider the orbits or curves travelled by planets as circles.

Let us take a piece of chalk tied to a piece of string and swing it around. You will notice that the chalk travels in a circle. Why? The force of the string constantly pulls the chalk in. This same idea can be applied to the Sun and the planets. If the Earth is at E, it will go along the line AB unless pulled in. When pulled in, it will be at C. It then tends to go along the tangent DG. When pulled in again, it will be at F. What force





is there, questioned Newton, in a body like the sun which will keep a body constantly moving around it in a circle?

Newton spent much time in thinking this over. In a world of no forces, a body or planet would do one of two things. It would be at rest, not move at all; or if it did move, it would go in a straight line, with a constant speed. The planet would do either thing because no forces would be working on it. Let us think of our railroad train in this connection. It may be at rest in the case when it is in the station or railroad yards. But after it has started and is moving at a constant speed, say forty miles an hour without changing its speed to fifty or thirty, the force exerted by the engine is for overcoming resistances that oppose the motion. These resistances are the friction of the wheels and bearings as they move and the impact of the air. If it were not for these, the train would keep its speed without further help from the engine. This is Newton's first law.

I. Every body continues in its state of rest or of moving with constant velocity in a straight line unless acted upon by some external force.

Now if a body is at rest and force is applied, (before, you remember, we had no forces) the body will move according to the force either fast or slowly. It is just like pushing a cart. If you press real hard, the cart will go fast. If you exert little energy or force, it will move slowly. This is Newton's second law.

II. The amount of acceleration or motion of a body depends upon the size of the force.

Let us look to our Sun and Earth. The speed of the Earth in its circular orbit will depend upon the force which the Sun expends. The acceleration required to keep the Earth in its circular path is $\frac{v^2}{r}$ where v is the speed of the Earth or planet and r is the radius of circle.

From the second law the force = the mass times the acceleration i.e. F = ma

$$\text{or } F = m\frac{v^2}{r}$$

The speed is v and the planet goes around the circular path which is $2\pi r$. The period or the time it takes a planet to go around the circle once is

$$T = \frac{2\pi v}{r} \qquad \left(\text{time} = \frac{\text{distance gone}}{\text{speed}}\right) \quad \text{or } v = \frac{2\pi r}{T}$$

From above we have $F = \frac{mv^2}{r}$ Substituting for v^2 the equation becomes

$$F = \frac{4\pi^2 r^2 m}{T^2 r} = \frac{m\,4\pi^2 r}{T^2}$$

This is the force then that is required to keep a body moving in a circular path.

But Kepler's third law said

$$F = \frac{r^3}{T^2}$$

Newton then said that his work must be wrong because it didn't agree with Kepler's.

How can we make the two laws agree? The first thing which we must recall is that Newton's law is for planets in circular orbits; Kepler's law for planets in elliptic orbits.

For a number of years Newton placed his computations aside. According to legend, the falling of an apple in his

garden renewed his interest. There must be some attraction between the apple and the Earth, he thought. What is this attraction? How are the two objects related? Finally, the law of gravitation was stated by him in 1682: -

Every particle of matter in the universe attracts every other particle with a force which is proportional to the product of their masses and which varies inversely as the square of the distance between them.

$$F = K\frac{Mm}{r^2}$$

If we have two planets of masses M and m, the force of attraction between them is equal to their product divided by the square of the distance between them.

If we let the two forces equal one another

$$F = m\frac{4\pi^2 r}{T^2} = K\frac{Mm}{r^2}$$

$$\frac{4\pi^2 r}{T^2} = \frac{KM}{r^2}$$

$$\frac{r^3}{T^2} = \frac{KM}{4\pi^2}$$

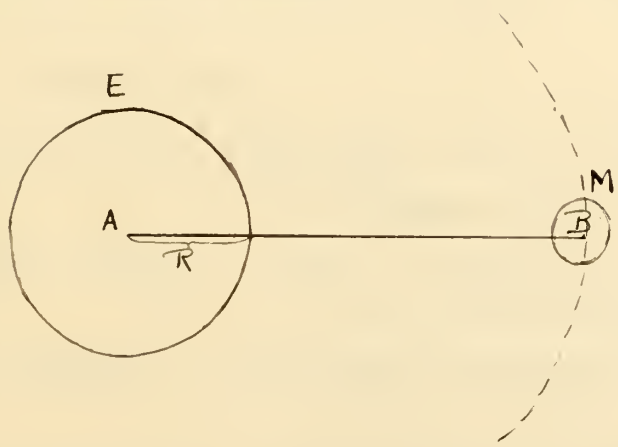
which equals Kepler's law for K is a constant body. Checking by using ellipses we get the same results.

The case of a falling stone is similar to that of the falling apple. It is an example of a force exerted by the earth on the stone. As far as the planets are concerned, the force necessary to keep them in their path is exerted by the sun.

The next step was for Newton to consider whether his law held in all cases. Would it account for the motion of the Moon around the Earth? If the Earth attracts a stone, it must surely attract the Moon.

The Earth's attraction for the Moon falls off as the square of the distance between them. The distance of the Moon from the Earth's center is sixty times the radius of the Earth. Hence the attraction exerted by the Earth on the Moon will be $\frac{1}{60 \times 60}$ of the attraction it will exert on a body at its own surface. This is evident for a body at the Earth's surface is $\frac{1}{60}$ of the distance of the Moon to the Earth's center.

Let us compute the motion or acceleration of the Moon and see whether it is $\frac{1}{(60)^2}$ or $\frac{1}{3600}$ of the acceleration of gravity of the Earth.



$F = \frac{4\pi^2 r}{T^2}$ = acceleration

R = 4000 miles

r = AB = 60 x 4000

The time it takes the Moon to go around the Earth is 27.322 days.

----Moon's Orbit.

E = Earth
M = Moon
R = radius of E.

$$a = \frac{4 \times \left(\frac{22}{7}\right)^2 \times 5280 \times 60 \times 4000}{(27.322 \times 24 \times 3600)^2}$$

a is now expressed in feet per second per second. There are 5280 feet in a mile so r is multiplied by it. There are 24 hours in a day, 60 minutes in an hour, and 60 seconds in a minute. $\pi = \frac{22}{7}$ or 3.1414 may be used. a = .00898 feet per second per second. If we multiply a by 3600, the acceleration of gravity is 32.3 feet per second per second which is that of the earth. 32 is commonly used as the

acceleration of gravity differs slightly depending upon whether one is situated near or far from the poles.

Thus we see that the law of gravitation, stated by Newton, does hold true for the motion of the Moon around the Earth. In a similar fashion, it has been proven to be true for all the planets travelling in ellipses with respect to the sun; for the planets with respect to their moons or satellites, and also in the case of binary stars, two stars which revolve one about the other. Newton, incited by Kepler's laws, worked out the proof for the gravitation theory which had bothered scientists and mathematicians for years.

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Many units similar to this may be worked out satisfactorily using the suggested books for collateral reading and others depending upon the unit selected.

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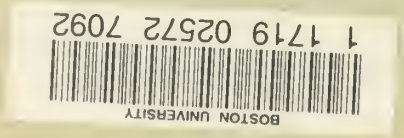
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